A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

1

LA-UR -84-1996

NOTICE
PORTIONS OF THIS REPORT ARE ILLEGIBLE. It has been reproduced from the best available copy to permit the broadest possible availability.

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

LA-UR--84-1996

DE84 014027

TITLE: CEPHEID EVOLUTION

AUTHOR(S): Stephen A. Becker

SUBMITTED TO: IAU COLLOQUIUM ON CEPHEID VARIABLES

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

FORM NO 836 R4

CEPHEID EVOLUTION

S. A. BECKER
University of California, Los Alamos National Laboratory,
Los Alamos, NM 87545 USA.

Abstract. A review of the phases of stellar evolution relevant to Cepheid variables of both Types I and II is presented. Type I Cepheids arise as a result of normal post-main sequence evolutionary behavior of many stars in the intermediate to massive range of stellar masses. In contrast, Type II Cepheids generally originate from low-mass stars of low metalicity which are undergoing post core helium-burning evolution. Despite great progress in the past two decades, uncertainties still remain in such areas as how to best model convective overshoot, semiconvection, stellar atmospheres, rotation, and binary evolution as well as uncertainties in important physical parameters such as the nuclear reaction rates, opacity, and mass loss rates. The potential effect of these uncertainties on stellar evolution models is discussed. Finally, comparisons between theoretical predictions and observations of Cepheid variables are presented for a number of cases. The results of these comparisons show both areas of agreement and disagreement with the latter result providing incentive for further research.

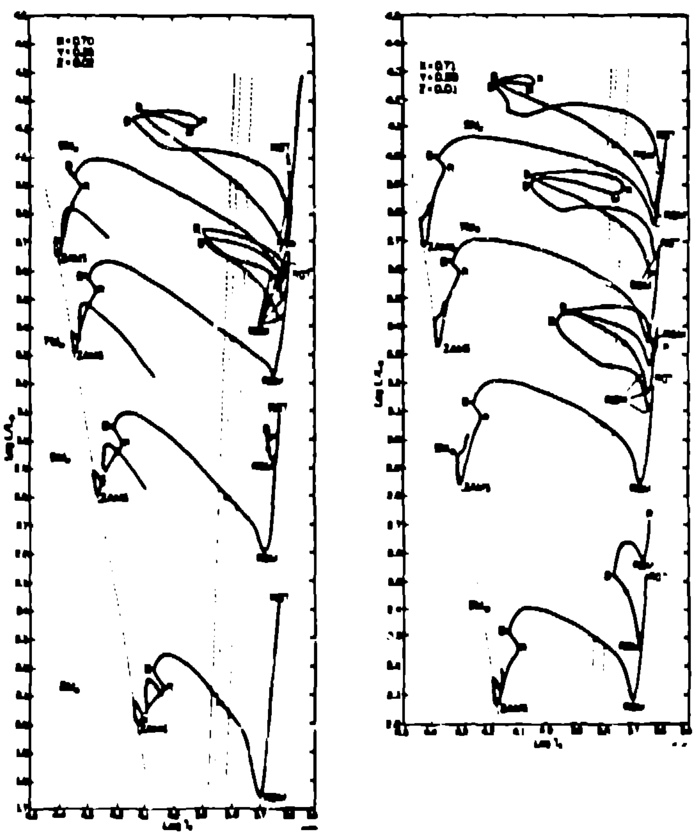
1 GENERAL OVERVIEW OF STELLAR EVOLUTION MODELS IN REFERENCE TO CEPHEID VARIABLES.

1.1 The Population I Picture

For purposes of discussion in this section, attention will be directed to results obtained from models of single, non-rotating stars. Population I stars are arbitrarily defined to be those for which $Z > 0.005$. Figure 1a shows the evolutionary tracks in the H-R diagram for a number of intermediate-mass stars of composition $(Y,Z) = (0.28, 0.02)$. Intermediate-mass stars are those which ignite He non-degenerately but following core He exhaustion develop electron degenerate carbon-oxygen cores. For the composition depicted in Figure 1a, such stars span the range of approximately 2.25 to 9 $M_\odot$. Stars of mass greater than approximately 9 $M_\odot$ do not develop degenerate carbon-oxygen cores and are called massive stars. Finally, stars of mass less than approximately 2.25 $M_\odot$ are called low-mass stars and these objects develop electron degenerate He cores prior to core He ignition.

In Figure 1, the boundaries of the Cepheid instability strip (as determined by the calculations of Iben & Tuggle, 1975) are represented by three parallel dashed lines which are, going from left to right, the first harmonic blue edge, the fundamental blue edge and the fundamental red edge. Standard pulsation theory argues that whenever a star's evolutionary track lies within the Cepheid strip, the star is unstable to surface pulsations and the star should be recognizable as a Cepheid variable. Observations confirm that the majority of stars in the instability strip are indeed Cepheid variables although some exceptions exist (see e.g. Eggen, 1983 and Bidelman, this conference). The boundaries of the Cepheid instability strip extend to the domains of the massive and low-mass stars not shown in Figure 1.

FIGURE 1. Evolutionary H-R Diagrams of models with Y = 0.28. In (a) Z = 0.02 and (b) Z = 0.01.



Stellar evolution calculations show that the progenitors of Cepheid variables are main sequence stars, and that to become a Cepheid a star must be in a post core hydrogen-burning phase. Figure 1 shows that a given intermediate-mass star can cross the Cepheid instability strip (depending on its mass or composition) once, thrice, or five times. Massive stars in the range of 9 to 20 $M_{\odot}$ (depending on the mass-loss rate) can also experience up to three crossings (see e.g. Sreenivasan & Wilson, 1978). Stars more massive than 20 $M_{\odot}$ (see e.g. Brunish & Truran, 1982a) as well as stars in the low-mass range (see Mengel et. al, 1979) can undergo at best one crossing. The first crossing occurs for all but the most massive stars during the hydrogen-burning shell phase as the star evolves to cooler temperatures in the H-R diagram on its way to becoming a red giant. The time scale for the first passage through the instability strip ranges from a few x 10^6 yr to about 10^3 yr with the lifetime decreasing with increasing mass. This time scale is approximately on the order of a thermal (Kelvin-Helmholtz) time scale. For example, the 3 $M_\odot$ model of Figure 1a takes about 2 x 10^5 yr to evolve from the fundamental blue edge to the fundamental red edge while a 9 $M_\odot$ model of the same composition takes about 4 x 10^3 yr.

There are effective limits to prevent stars from becoming Cepheids at both the high and low extremes of the stellar mass range. For stars of low enough mass like the sun, the main sequence surface temperatures are cooler than the red edge of the instability strip and consequently, their normal evolutionary tracks do not intercept the instability strip. In fact, low-mass stars which do intersect the instability strip are recognized as a separate class of variables called the δ Scuti stars. Although the boundary between Cepheids and δ Scuti stars is somewhat indistinct, Cepheid variables in practice originate from stars of at least intermediate mass. At the other extreme, stars more massive than about 40 $M_\odot$ appear to lose mass so prodigiously that instead of evolving into red giants their evolutionary tracks reverse before the instability strip and evolve blueward into the domain of the WR stars (deLoore, 1980). This effect limits the brightest Cepheids to periods of no more than about 200 days.

The second crossing of the instability strip occurs during the core helium-burning phase as a star evolves to higher temperatures on the first blue loop in the H-R diagram. Only intermediate-mass stars and massive stars of about 9 - 20 $M_\odot$ exhibit blue loops in their H-R diagrams. The first blue loop arises due to a complicated interplay between the X abundance profile left by the former hydrogen-burning core, the hydrogen-burning shell, the maximum depth reached by the convective envelope during the star's first ascent of the red giant branch, and the nature of stellar envelope solutions (see Schlesinger, 1977 for a review). When the tip of the first blue loop is tangent to the blue edge of the instability strip, passage through the instability strip is driven on a nuclear time scale lasting over several x 10^6 yr. For the composition depicted in Figure 1a, this condition would occur for a model of about 6 $M_\odot$. The lifetime of the second crossing decreases with increasing stellar mass approaching a thermal time scale at larger masses due to the fact that a significant portion of the core helium-burning phase is spent in the vicinity of the first blue loop tip and that as more massive stars are considered the temperature of the blue loop tip increases. In Figure 1a the lifetime of the second crossing is for example about 2.6 x 10^5 yr for the 7 $M_\odot$ model and 3 x 10^3 yr for the 9 $M_\odot$ model.

When it occurs, the second crossing of the instability strip is almost always the longest lived. (It is only in the case of the more massive blue-looping models where all crossings are effectively driven on a thermal time scale that this rule is no longer valid). Consequently, most Cepheids observed are stars undergoing the second passage of the instability strip. In addition, the occurrence of the first blue loop provides an effective lower mass limit to the observed distribution of Cepheids. Only stars which are able to evolve to a high enough temperature to intercept the instability strip during their first blue loop are generally seen as Cepheids. Stars whose first blue loop does not intercept the instability strip can only be observed as Cepheids during their first passage of the strip and these stars due to their shorter lifetimes probably make up only about 10% of the total observed Cepheid population.

The third passage of the instability strip can occur under two different conditions. The most common of the two takes place on the first blue loop near the end of the core helium-burning phase as the star evolves back to the red giant branch. Such is the case for the 7 $M_{\odot}$ models of Figure 1. Massive stars which intercept the Cepheid instability strip only once are also at a similar point in their internal evolution, i.e., they are also near the end of their core helium-burning phase, however, unlike the intermediate-mass stars, they are evolving to the red giant branch for the first time.

The lifetime for the third passage of the instability strip when it occurs near the end of the core helium-burning phase can be as large as just over 10^6 yr for the case of a model whose blue loop tip is tangent to the blue edge of the instability strip. The lifetime of this crossing then deceases with increasing mass until it no longer takes place and the second option for the third crossing is instead in effect. The 7 $M_{\odot}$ model of Figure 1e is an example of the first option for the third crossing and this passage has a lifetime of about 5×10^4 yr which is about one fifth as long as the lifetime of the second crossing but nearly 10 times as long as the lifetime of the first crossing. Generally, the the first option for third passage of the instability strip is the second longest crossing.

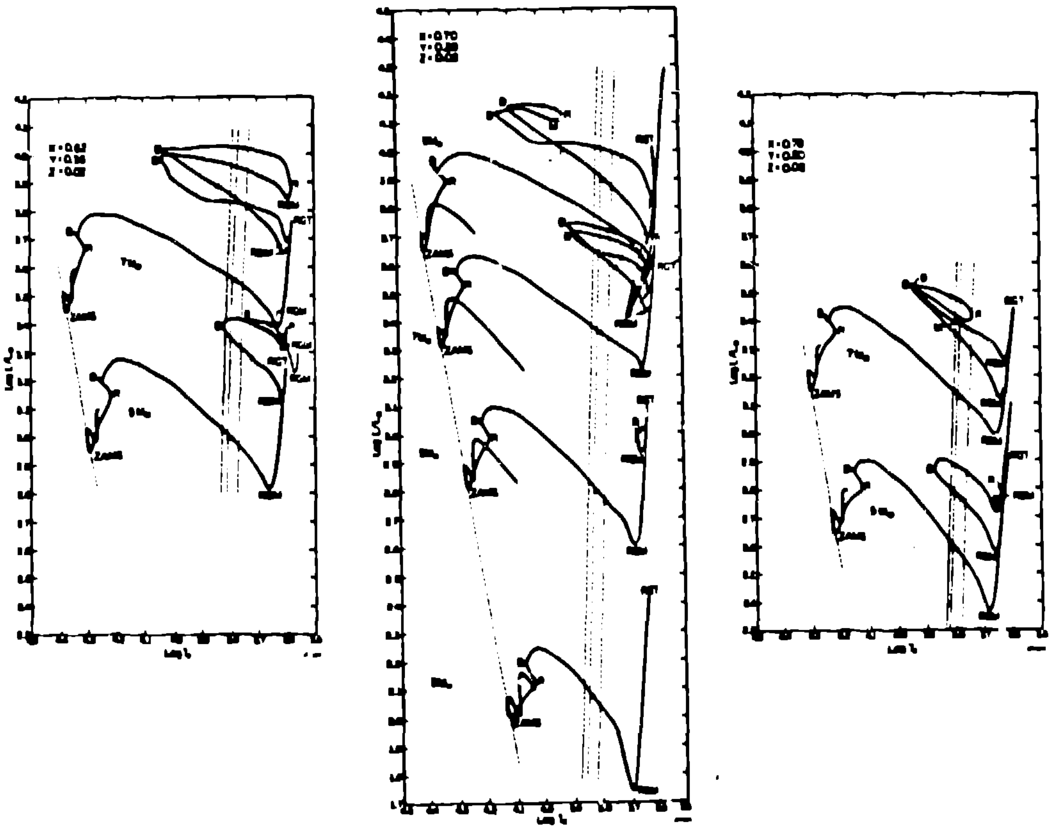
The other condition under which a third crossing of the instability strip can occur takes place after core helium exhaustion during the helium-burning shell phase. Such is the case for the 9 $M_{\odot}$ models of Figure 1 and other more massive models which experience blue loops in the H-R diagram. In this case evolution takes place so rapidly in the stellar interior that the model is unable to cross the instability strip until after the helium-burning shell has established itself as the primary energy source for the star. As a result, the second option for the third crossing is the final passage of the instability strip for the stars which experience it. This type of crossing takes place rather rapidly on the order of a thermal time scale lasting approximately 10^3 yr.

Under certain circumstances a second blue loop may occur in the H-R diagram which does intercept the instability strip allowing for two additional crossings of the Cepheid strip. Hoppner et. al. (1978) and Becker (1981b) show that this additional loop is due to a complicated interaction between the contracting helium exhausted core, the helium-burning shell and the nature of the stellar envelope solutions. The lifetime of either the fourth or fifth crossing is roughly the same being potentially as large as a few x 10^5 yr for the case where the tip of the second blue loop is tangent to the blue edge of the instability strip. The lifetimes of these two crossings then decrease rapidly with increasing mass until for the more massive models the second blue loop takes place entirely to the left of the instability strip. For example for the 7 $M_\odot$ model of Figure 1a, the lifetimes of the fourth and fifth crossings are each approximately 10^4 yrs. Both the fifth crossing of Cepheid strip and the second condition for the occurrence of the third crossing of the Cepheid strip (see the 9 $M_\odot$ models in Figure 1) take place at the same evolutionary phase, i.e., during the helium-burning shell phase.

To summarize, Population I stars observed to be in the Cepheid instability strip can be undergoing, depending on the circumstances, the hydrogen-burning shell phase, the core helium-burning phase (with an active hydrogen-burning shell) or the helium-burning shell phase. When multiple crossings of the instability strip are possible, the most likely phase to be encountered is the core helium-burning phase with the second crossing being the longest lived of all the passages and the third crossing being the second longest. The cumulative lifetime of the other three crossings (when they occur) is generally small when compared to time spent during the second and third crossings. It is only for the more massive cases (like the 9 $M_\odot$ models in Figure 1) where all crossings of the strip take place on a thermal time scale that the lifetimes of the various crossings became comparable to each other. For stars of sufficiently small or large mass only one crossing of the instability strip is possible. In our galaxy, Cepheids having non-harmonic periods less than 3 days are most likely due to stars which make only one crossing of the Cepheid strip during their hydrogen-burning shell phase. At the other extreme, Cepheids having periods greater than about 30 days are most likely due to massive stars making their single passage of the strip near the end of their core helium-burning phase. The bulk of the Cepheids observed in the Galaxy, however, have periods between these two extremes and the vast majority of these stars should be in the core helium-burning phase. The rarest type of Cepheid would be one which is undergoing the fourth or fifth crossing of the instability strip.

Finally, it should be noted that the evolutionary behavior of a stellar model of a given mass varies as the initial composition is changed (see Becker et. al., 1977 and Becker 1981a for an extensive discussion). Figure 1b shows how models of the same masses as used in Figure 1a behave when their metalicity has been reduced to Z = 0.01. For this case, stars having a mass as small as about 4 $M_\odot$ become Cepheids during the core helium-burning phase as opposed to about 6 $M_\odot$ being the lower limit when Z = 0.02. The effect of changing the helium content is shown in Figure 2 where much larger changes in Y are required to produce changes of a similar size as those produced by much smaller changes in Z. By taking into account the composition dependence on a model's evolutionary behavior one can show that the average mass of a Cepheid is about 6 $M_\odot$ for the Galaxy, 4.5 $M_\odot$ for the LMC, and 3.5 $M_\odot$ for the SMC. In general for a model of fixed mass, reducing Z or increasing Y acts to make nearly all evolutionary phases of a model both hotter and brighter.

FIGURE 2. Evolutionary H-R diagrams of models with Z = 0.02. In (a) Y = 0.36, (b) Y = 0.28, and (c) Y = 0.20.



In any case, the progeny of Cepheid variables are later seen as red giants and red supergiants. Intermediate-mass stars evolve onto the asymptotic giant branch (AGB) and undergo helium-shell flashes during the double shell (hydrogen- and helium-burning) phase. The majority of these stars lose their massive envelopes through mass loss and evolve into white dwarfs. A small fraction of the most massive intermediate-mass stars may experience degenerate ignition of the central carbon and become carbon-deflagration supernovae. Massive stars, in contrast, evolve into red supergiants that undergo all the remaining phases of nuclear burning before possibly becoming Type II supernovae.

1.2 The Population II Picture

If single non-coalesced stars of Population II composition ($Z < 0.005$) having masses greater than 2 $M_\odot$ still existed today, such objects would undergo the same behavior as discussed in the previous section and consequently, some of these stars would mimic the behavior of Population I Cepheids. Unfortunately except perhaps in the SMC, no examples of this type of star currently exist although such stars must have existed in the past. Nonetheless as described in the review article by Wallerstein & Cox (1984), Population II Cepheids do exist but they arise due to circumstances that are different from those experienced by Population I Cepheids. Population II Cepheids obey a different period luminosity relation from that of the Population I Cepheids. As a class Population II Cepheids lie in the H-R diagram between the RR Lyrae variables at low luminosities and the RV Tauri and long period variables at high luminosities. Population II Cepheids are broken into three main categories which are BL Herculis (BL Her) stars, the W Virginis (W Vir) stars, and the anomalous Cepheids.

Population II stars having an age near that of the universe can initially be no more massive than 0.8 $M_\odot$ in order to be observed today. Such a star ignites helium under degenerate conditions which leads to the core helium flash and evolution to quiescent core helium-burning on the horizontal branch (see e.g. Figure 1 of Despain, 1981). A portion of the horizontal branch intercepts the instability strip and any stars located at this junction should behave as RR Lyrae variables (see Iben, 1974a for a review of the horizontal branch phase). When helium is exhausted in the center of a horizontal branch star, the star evolves upward in the H-R diagram onto the suprahorizontal branch where the thick helium-burning shell phase is established (see e.g. Iben & Rood, 1970, and Sweigart & Gross, 1976). If while on the horizontal branch the star is located either inside or to the left of the instability strip in the H-R diagram, the post-horizontal branch evolution will cause the star's evolutionary track to intercept the instability strip at which point the star should behave as a BL Her variable (Gingold, 1974 and Iben, 1974b) with a period between 1 - 5 days. Depending on the mass and composition of the star, the evolutionary track may intercept the instability strip more than once with three crossings being the maximum number (see e.g. Figure 7 of Iben & Rood, 1970). When three crossings occur, the third crossing has the longest duration. Unlike Population I Cepheids, BL Her stars originate from a very narrow mass range of about 0.6 $M_\odot \pm 0.05$ $M_\odot$. The total lifetime of this phase of variability lasts up to several x 10^6 yr due to the evolution proceeding on a nuclear-burning time scale.

Eventually a low-mass Population II star evolves off the supra-horizontal branch to the AGB where the hydrogen-burning shell re-establishes itself and the double burning shell phase begins. When the hydrogen envelope is nearly exhausted ($<5 \times 10^{-3}$ $M_\odot$) Gingold (1974) and Schonberner (1979) find that as a result of the last helium shell flash on the AGB one or two loop-like excursions from the AGB are possible for certain models. These excursions can intercept the instability strip causing a star to become a W Vir variable having a period of roughly 10 to 50 days. Even if no loop-like excursion occurs for a particular model, once the hydrogen envelope is essentially exhausted ($< 3 \times 10^{-3}$ $M_\odot$) the star is forced to evolve off the AGB across the H-R the diagram toward the realm of the white dwarfs (see e.g. Figure 3 of Iben, 1982). Such a track results in one guaranteed passage of the instability strip.

Again as is the case for the BL Her stars, the W Vir stars originate from a narrow mass range which is about 0.6 $M_\odot \pm 0.1$ $M_\odot$. The lifetime of this phase of variability can range from about 10^4 yr for the single crossing case or the most optimum loop-like excursion to a few x 10^2 yr for the most transient of the loop-like excursions. Observations of period changes of W Vir stars should help to determine if these loop-like excursions actually occur.

Finally, in dwarf spheroidal galaxies there is another type of Population II Cepheid observed which are known as the anomalous Cepheids because these Cepheids are more luminous at a given period than Cepheids found in globular clusters. Hirshfeld (1980) has shown that extremely metal poor stars ($Z < 5 \times 10^{-4}$) of mass 1.3 to 1.6 $M_\odot$ also lie on something like a horizontal branch during their core helium-burning phase unlike their more metal rich Population II counterparts. This horizontal branch for more massive low-mass stars occurs at a higher luminosity than the traditional horizontal branch, and portions of this new horizontal branch intercept the instability strip (see e.g. Figure 1 of Hirshfeld, 1980). Stars located at this intersection are found to match the observed properties of anomalous Cepheids.

Without invoking recent star formation, the initial mass of stars in Population II systems must be 0.8 $M_\odot$ or less. In order to form stars in the range of 1.3 to 1.6 $M_\odot$ in dwarf spheroidal galaxies, Hirshfeld (1980) argues for coalescence of two stars via binary mass transfer. Wallerstein and Cox (1984) reach the same conclusion based on their analysis of the pulsational masses of anomalous Cepheids. Once formed out of two stars, the coalesced star can have a lifetime in the instability strip of several x 10^7 yr.

To summarize, Population II Cepheids originate from a much narrower and smaller range of masses than Population I Cepheids. Most Population I Cepheids are in the core helium-burning phase while only the anomalous Population II Cepheids of very metal poor systems are undergoing this evolutionary phase. Most Population II Cepheids (i.e., the BL Her and the W Vir stars) are in their post core helium-burning phases prior to their becoming white dwarfs.

2 THEORETICAL UNCERTAINTIES IN THE STELLAR EVOLUTION CALCULATIONS

A theoretical model is only as good as the physics going into it, and therefore uncertainties in how best to model a physical process like convection or uncertainties in the input physics like the nuclear reaction rates can lead to uncertainties in the evolutionary results. In this section a brief discussion of the various modeling problems is presented in terms of how these uncertainties may affect the theoretical predictions for Cepheid variables.

2.1 Convective Overshoot and Semiconvection

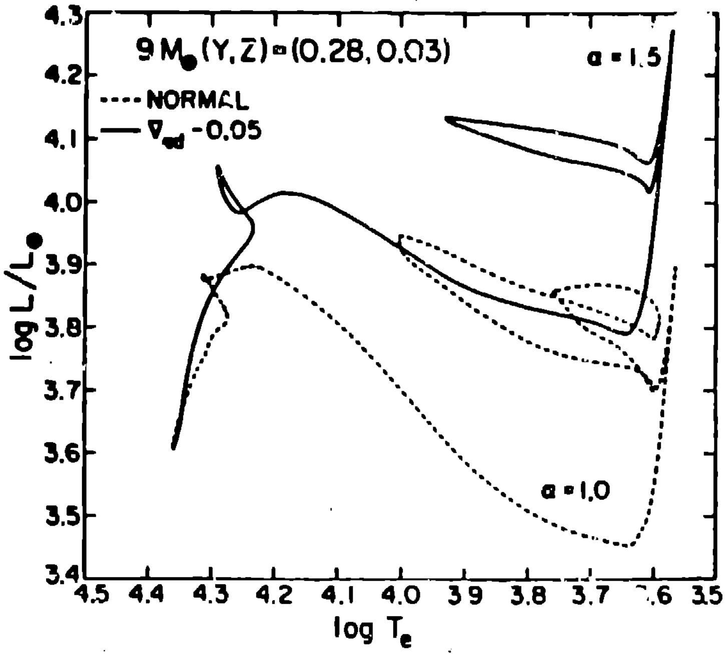
Convective overshoot occurs at the boundary of a formally convective region where the kinetic energy of a convective element carries it a finite distance into a region which is formally stable against convection. Mixing may arise between the convective region and a part of its neighboring radiative region due to the finite decay time of overshooting convective elements and the introduction of new material into the radiative region from the overshooting convective elements causing part of the radiative region to become convectively unstable. Semiconvection occurs in a region of variable chemical composition that is marginally unstable to convection. In such a case convective mixing is only able to partially mix the region before convective stability is established. When a semiconvective region exists just outside a convective region mixing may occur between the two regions.

With regard to Cepheid evolution, the convective cores of the core hydrogen-burning and the core helium-burning phases are the most sensitive to any uncertainties in modeling convective overshoot and semiconvection. Convective overshoot occurs throughout the existence of a convective core while semiconvection tends to appear only outside the helium-burning convective core (see, however, Brunish & Truran, 1982a,b for a description of how massive stars behave). Both effects bring more fuel to the convective core resulting in a longer life for a given core burning phase.

Maeder & Mermilliod (1981), Matrika et. al. (1982), and Huang & Weigert (1983) discuss the effect convective overshoot has during the core hydrogen-burning phase on the later evolutionary behavior of a model. Compared to models where this effect is neglected, models which include the effect of overshoot are brighter during their core helium-burning phases and their first blue loops in the H-R diagram (when they occur) tend to be somewhat smaller. Figure 3 (from Becker & Cox, 1982) shows this effect for a 9 $M_{\odot}$,(Y,Z) = (0.28, 0.03) model.

Robertson (1972), Robertson & Faulkner (1972) and Renzini (1977), discuss the effect of including convective overshoot and semiconvection during the core helium-burning phase. Compared to models where these effects are neglected, models which include these effects have longer core helium-burning lifetimes and their first blue loops (when they occur) may be lengthened.

FIGURE 3. The theoretical H-R diagram for a 9 $M_{\odot}$, (Y,Z) = (0.28,0.03) model without convective overshoot (dashed line) and with convective overshoot (solid line).



Although there is general agreement about the existence of convective overshoot and semiconvection, the degree to which these effects occur is still an open topic of discussion. One hope of determining the extent of these effects is to calibrate the models by using the observed color-magnitude diagrams of star clusters (Maeder & Mermilliod, 1981), however, observational uncertainties then enter the picture. In any case when these effects are included in stellar evolution models, the predicted evolutionary mass for a Cepheid variable of a given luminosity is smaller than the case where these effects are neglected. Matraka et. al. (1982), for example, argue that this reduction may be as large as 15%, a result which is not inconsistent with predictions of other studies.

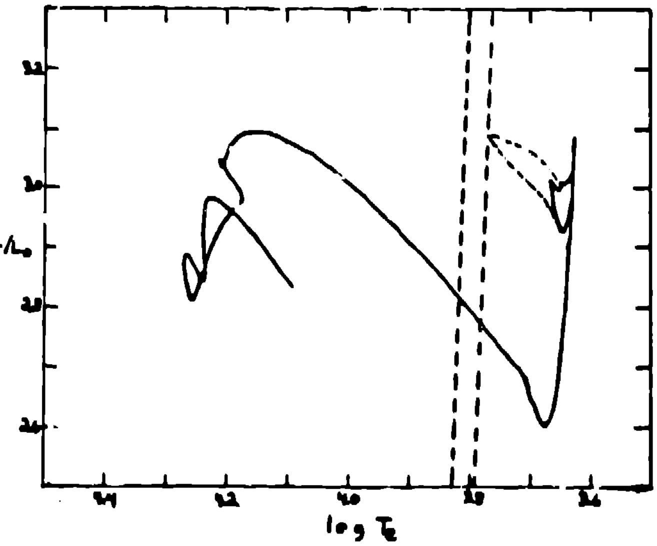
2.2 Nuclear Reaction Rates

A study of the works of Fowler et. al. (1967, 1975, FCZII) and Harris et. al. (1983, HFCZ III) show that many nuclear reaction rates are not known precisely and that the published rates have changed over the years. With regard to Cepheid variables, the most relevant nuclear reaction rates are those involving the burning of hydrogen and helium. Fortunately, the important hydrogen-burning rates have changed little over the years, however, such is not the case for the helium-burning reactions particularly, the $^{12}C(\alpha,\gamma)^{16}O$ rate. Iben (1972) was the first to show how uncertainty in the reduced width term of this reaction rate would affect the evolutionary behavior of stellar models that become Cepheids.

Since the $^{12}C(\alpha,\gamma)^{16}O$ rate did not change between FCZ II and HFCZ III it began to appear that this rate was on much firmer ground than it was previously. However, Kettner et. al. (1982) made a new measurement of this rate and concluded that the rate should be approximately five times greater than the rate given in FCZ II! Fowler (1984) has restudied the problem from the Caltech data base and has concluded that the increase should be instead about a factor of three greater than the FCZ II rate. Settlement of this controversy will have to await new measurements of the $^{12}C(\alpha,\gamma)^{16}O$ rate.

Figure 4 shows the appearance of the core helium-burning phase in the H-R diagram for a 5 $M_\odot$,(Y,Z) = (0.28, 0.02) model as a result of using both the FCZ II rate and Fowler (1984) $^{12}C(\alpha,\gamma)^{16}O$ rate. With the new reaction rate the lifetime of the core helium-burning phase is lengthened by about 5%, and the blue loop extends to higher temperatures. The lengthening of the blue loop is only noticeable in models for which the loop was small to begin with. One important effect of this change is that the minimum mass of a star that becomes a Cepheid during core helium-burning phase would drop going in the case of a (Y,Z) = (0.28, 0.02) composition from about 6 $M_\odot$ to 5.1 $M_\odot$ if the Fowler (1984) rate is adopted.

FIGURE 4. The theoretical H-R diagram for a 5 $M_\odot$, (Y,Z)=(0.28,0.02) model as a result of using the FCZ II $^{12}C(\alpha,\gamma)^{16}O$ rate (solid) and the Fowler (1984) $^{12}C(\alpha,\gamma)^{16}O$ rate (dash-dot). The parallel dashed lines mark the fundamental boundaries of the instability strip.

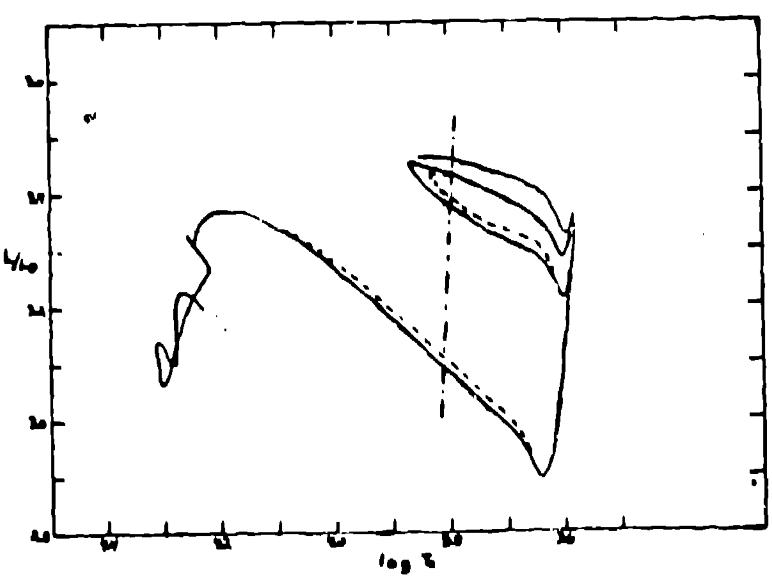


2.3 Opacities and Stellar Atmospheres

The studies of Fricke et. al. (1971) Henyey et. al. (1965), and Johnson & Whittaker (1975), to name a few, show that changing the opacity used either in the stellar interior or the stellar atmosphere can have a noticeable effect on a stellar model's evolution track in the H-R diagram. For Cepheid variables the most important question was whether the Carson opacities (see e.g. Carson & Stothers, 1976) or the Los Alamos opacities (see e.g. Cox & Tabor, 1976) were the most accurate. The primary difference in behavior between the two sets of opacities is that the Carson opacities have noticeably larger contributions from He and CNO atoms to the opacity in the temperature range of 6.5 > log T > 5.4. The question remained unresolved until last year, when Carson visited Los Alamos and a direct comparison was made between the two methods of calculation and an error was discovered in the Carson code which produced the larger opacities.

With this controversy resolved, the effect of the remaining uncertainties in the opacities or the modeling of stellar atmospheres on the evolutionary behavior of Cepheids is relatively small. For example, Simon (1982) has suggested that increasing the opacities due to heavy elements by a factor of 2 to 3 in the temperature range of 10^5 K to 2×10^6 K could help to remove the mass anomalies encountered with the double-mode and bump Cepheids. Figure 5 shows how the evolutionary track of a 6 $M_\odot$,(Y,Z) = (0.28, 0.02) model changes in going from the standard opacities to one similar to that suggested by Simon. The overall differences are small and consequently, from the stellar evolution standpoint, the changes advocated by Simon are potentially

FIGURE 5. The theoretical H-R diagram for a 6 $M_\odot$, (Y,Z)=(0.28,0.02) model using standard Los Alamos opacities (solid) and opacities in which the heavy element contribution is doubled for $0.1 < T_6 < 0.5$ (dashed). The fundamental blue edge of the instability strip is represented by the dash-dot line.



permissible. However, Magee et. al. (1984) point out that there is no reason to expect an uncertainty that large in the Los Alamos opacities.

2.4 Mass Loss

Lauterborn et. al. (1971) and Lauterborn & Siquig (1974) have studied the effect of mass loss during the red giant phase of intermediate-mass stars of extreme Population I compositions. They find that mass loss is able to suppress formation of blue loops in the H-R diagram if more than 10%, 13%, and 20% of the total stellar mass is lost respectively for 5 $M_{\odot}$, 7 $M_{\odot}$ and 9 $M_{\odot}$ models. Forbes (1968) and Lauterborn et. al. (1971) find that the blue loops remain suppressed until greater than 60% of the total stellar mass is lost. While the above results probably depend somewhat on the initial composition of the star, the implication is clear that a moderate degree of mass loss for intermediate-mass stars effectively reduces the number of crossings of the Cepheid instability strip from a maximum of five to only one. Fortunately, the observed mass loss rates (see, e.g., Lamers, 1981 and Riemers, 1975) indicate that very little mass is lost during the pre-Cepheid evolution for single intermediate-mass stars. Only low-mass stars during the red giant phase and massive stars during all their phases appear to lose a significant fraction of their mass.

For low-mass stars, mass loss plays a significant role in their evolution because without it these stars would not evolve onto the horizontal branch after core helium ignition. Without a horizontal branch it would not be possible to produce RR Lyrae stars, BL Her stars, and possibly anomalous Cepheids. For massive stars, mass loss increases with increasing luminosity so that above about 40 $M_{\odot}$ no star is able to evolve as red as the Cepheid strip. Mass loss when included in models of massive stars causes evolution to proceed at a lower luminosity than the case where it is not included (see e.g. Brunish & Truran, 1982a,b).

As noted in Lamers (1981) and Reimers, (1975) the mass loss rates are not precisely known and it is likely that a range of rates is possible for stars having the same mass and luminosity. For intermediate-mass stars the uncertainty in the mass loss rates is not of much consequence for Cepheid evolution. However, in case of massive stars, any underestimate of the mass loss rate will lead to an underestimate of the evolutionary mass for a Cepheid of a given luminosity.

2.5 Rotation and Stellar Evolution

Observation of the main sequence progenitor stars that evolve into Population I Cepheids shows that they rotate at speeds of 200 to 270 km/sec (see e.g. the review article of Slettebak, 1970). Observations of red giants (see e.g. Kraft, 1970) show, however, that these stars have small rotational velocities. These observations indicate that the surface angular momentum present during the main sequence is transferred to another location which most likely is the interior of the star. Clearly, since real stars have angular momentum, the neglect of rotation in most stellar evolution calculations is a deficiency. Problems arise, however, when one attempts to add rotation to a stellar model. Rotation destroys the one-dimensional symmetry of a stellar model and an accurate treatment of rotation requires two- or three-dimensional coding capability. Modern computers can accommodate codes of this complexity and efforts to develop such codes are beginning to take place. With some simplifications, however, rotating models can be studied with one-dimensional stellar evolution codes. Problems still confront any investigator using this latter approach, since the interior angular momentum distribution cannot be observed. One therefore needs to make a number of educated guesses as to what the distribution might be.

Despite the difficulties, Kippenhahn et. al. (1970), Meyer-Hofmeister (1972) and Endal & Sofia (1976, 1978, 1979) have investigated from a one-dimensional standpoint the influence of rotation on the evolutionary behavior of intermediate-mass stars. Their results provide a mixed picture. For example, they all agree that rotation lengthens the core helium-burning lifetime and that this increase can result in longer lifetimes for the second and third crossings of the instability strip. However, Endal & Sofia (1976, 1978) find that rotation increases the luminosity of the first blue loop in the H-R diagram, but reduces the temperature of the blue loop tip while the work of the others finds the opposite results. Due to these disagreements, it is clear that more work needs to be done before an accurate assessment of how neglect of rotation in most stellar models is affecting theoretical predictions.

2.6 Binary Evolution

Madore (1977) notes that about 27% of the Population I Cepheids are in binaries which have companions of similar mass. Sandage & Tammam (1969) point out that one binary system, CE Cas, consists of two Cepheids. It is apparent that a significant fraction of Cepheid variables, as is the case for most Population I stars, occurs in binary systems. If the separation between the binary components is sufficiently large (on the order of a few x 100 R_0), each star will be able to evolve through its core hydrogen- and helium-burning phases in much the same way as if each star were single. If, however, the separation is such that one star fills its Roche lobe during the course of its evolution, mass transfer will occur and the evolutionary behavior of the two stars will no longer be accurately described by single star evolutionary models.

Detailed evolutionary calculations of a binary system consisting of intermediate or massive stars have not yet been calculated for a wide variety of cases. A few general statements can, however, be made. The more massive star of the system, if it fills its Roche lobe as a red giant, will probably lose enough mass to prevent it from making any further crossings of instability strip. As a result, this star can be a Cepheid only prior to its becoming a red giant. The companion star will probably accrete mass as a result of the more massive component overflowing its Roche lobe. Having accreted mass, the companion star will later evolve in a manner different from that of a single star having the companion star's original or current total mass. As a result, if the companion star later evolves into the instability strip it will do so at a luminosity that is probably inconsistent with its mass based on single star evolution.

Binary evolution is clearly an area for future research. The scenario described in the second half of the last paragraph is only a general case and even more complicated situations can be imagined as a result of binary evolution.

3 THEORY AND OBSERVATION COMPARED

Stellar evolution models can be used to make predictions which can be tested by comparison with observations. Such comparisons provide a diagnostic on the current state of the theoretical models. Areas of disagreement between theory and observation are useful because they help to provide some insight on how theoretical models can be improved. The subject of comparing theory and observations is a topic worthy of a separate paper and as a result this subject will be only briefly discussed in this section.

3.1 Cepheid Masses

Using stellar evolution calculations, a mass-luminosity relationship can be made for Cepheid variables (see e.g. Becker et. al., 1977). Using pulsational theory, a luminosity-period-temperature relationship can be derived. These two relations can be combined to relate period to mass if the exact crossing of the instability strip is known. If the exact crossing is not known, the usual procedure is to assume that the star is undergoing the second crossing. The work of van Genderen (1983), for example, shows that these theoretical relations can be used very successfully with the observed behavior of Cepheids in the Magellanic Clouds.

Cox (1980), in his review article, describes six methods of theoretically determining the mass of Cepheid variables which are: 1) the evolutionary mass, 2) the theoretical mass, 3) the pulsational mass, 4) the bump Cepheid mass, 5) the double and triple mode Cepheid mass, and 6) the Wesselink radius mass. Cox (1980) notes that the evolutionary mass is normally in agreement with all the other methods except the bump Cepheid masses and the double and triple mode Cepheid masses.

Since no Cepheid is close enough to determine distance by trigonometric parallax, the most direct method of getting Cepheid masses is from binary orbits and the results have been mixed. Evans (1980) orignally found for the case of SU Cygni that the orbital mass is consistent with masses predicted by both stellar evolution and pulsation theory, however, this interpretation is now clouded because this system has been found to be a triple (Evans, this conference). Bohm-Vitense (1984), finds, for binary systems consisting of a blue companion and a Cepheid variable, that the Cepheids are overluminous if both stars are of the same mass. The discrepancy may, however, be due to mass transfer inside the system.

The agreement in the case of Cepheid variables between evolutionary masses and pulsational masses from all but the bump and double and triple mode Cepheids shows that stellar evolution calculations are, in general, qualitatively correct (using the current distance scale). The discrepancy with the bump and double and triple mode Cepheids is a problem which is still not understood, but it is an active topic of research. The agreement between evolutionary mass and pulsation mass would end, however, if the luminosity scale of Cepheid variables is revised downward as advocated by Schmidt (1984).

3.2 Period Changes

During evolution across the instability strip, the period of a Cepheid increases as the temperature decreases. As a result of this behavior, measurements of the rate of period change can help to determine which crossing of the instability strip is taking place. For intermediate-mass stars the first, third, and fifth crossings will have a positive rate of change, while the second and fourth crossings will show a negative rate of change. The magnitude of the rate of change, $\dot{P}$, provides further information in that stars undergoing the second or third crossing will in general be evolving at a slower rate than stars undergoing the other crossings (see e.g. Becker 1981b). Parengo (1956), Payne-Gaposchkin (1974), Fernie (1979, 1984) and Szabados (1983, 1984) have studied period changes in Cepheid variables. Payne-Gaposchkin (1974), Fernie (1984) and Szabados (1983, 1984) note that measurements of the periods changes of Cepheids sometimes show nonsecular changes or noise which makes the determination of $\dot{P}$ complicated. However, a large number of sufficiently clear measurements of $\dot{P}$ have been made by Fernie (1984) and Szabados (1983,1984) for them to conclude that the observed period changes are due to effects of stellar evolution. Observation of positive and negative values of $\dot{P}$ provides direct evidence for the existence of blue loops in the H-R diagram.

3.3 Surface Changes in the Chemical Composition

Cepheid variables of intermediate mass that are in the core helium-burning phase spent part of their previous evolution as red giants. As red giants, these stars underwent the first dredge-up phase in which convection mixes the outer layers of the star with a portion of interior that in the past has undergone nuclear reactions. Stellar evolution calculations (see e.g. Becker & Iben, 197.) predict that the surface abundance of ^{14}N should increase by a factor of two to three and that the surface abundance of ^{12}C should decrease by 30 to 40% as a result of the first dredge-up phase.

Luck & Lambert (1981) were the first to measure CNO abundances in Cepheid variables and their results showed abundance changes far in excess of that predicted by standard stellar evolution theory. Furthermore, the observations of Luck & Lambert (1981) showed that mixing must take place with matter so deep in the interior that it has undergone ON processing. Becker & Cox (1982) discussed the implications of how stellar evolution models would have to be altered to account for these observations. Iben & Renzini (1983) took the opposite approach and discussed what might be wrong with the interpretation of the observations. The controversy has apparently now come full circle when

Lambert (1984) stated that the observed O depletion in Cepheids was a result of a systematic error in the values used for log g. He now finds evidence only for CN processing as predicted by standard stellar evolution calculations.

3.4 Frequency Period Distributions of Cepheids

From the observations of hundreds to thousands of Cepheid variables in the Galaxy, M31, the LMC, and the SMC, detailed but not complete frequency period distributions can be made. The distributions all show a small number of short period varibles, a rapid increase in the numbers of Cepheids at some key period and an exponential-like decline in the number of variables for larger and larger values of the period (see Figures 7 and 8 of Becker et. al. 1977). The distribution for the Galaxy and M31 are very similar, while those for the LMC and the SMC show a greater spread and the peak in their distributions takes place at a noticeably smaller value for the period.

The sudden rise in the frequency period distribution provides strong evidence for the first blue loop occurring in the H-R diagram for helium burning intermediate-mass stars. The peak should arise from stars in which the tip of the blue loop is tangent to the fundamental blue edge. Becker et. al. (1977) show that a galactic model based on stellar evolution calculations which involves a spread in the chemical composition and reasonable values for birthrate function can reproduce many of the features of the observed frequency-period distributions. The reason for the different peaks in the distributions for the Galaxy, the LMC, and the SMC is a result of having the average chemical composition change as one goes from the Galaxy, to the LMC, and to the SMC.

3.5 Star Clusters

Unlike the case for the Galaxy, a number of star clusters in the LMC and the SMC have a high population density and are the right age to show significant numbers of Cepheids and stars in the core helium-burning phase. Arp (1967) was the first to note this behavior in the case of NGC 1866. These clusters can be modeled with synthetic clusters constructed from stellar evolution models. For the case of NCG 1866 as shown in Figure 6, the observed color-magnitude diagram provides visual proof of the predictions of stellar evolution theory because the diagram looks like a blurred stellar evolution track with the Cepheids appearing at the blue loop tip. Becker and Mathews (1983) have analyzed the data for NGC 1866 with synthetic cluster models and have shown its age to be about 86×10^6 yr and its composition to be about (Y,Z) = (0.273, 0.016).

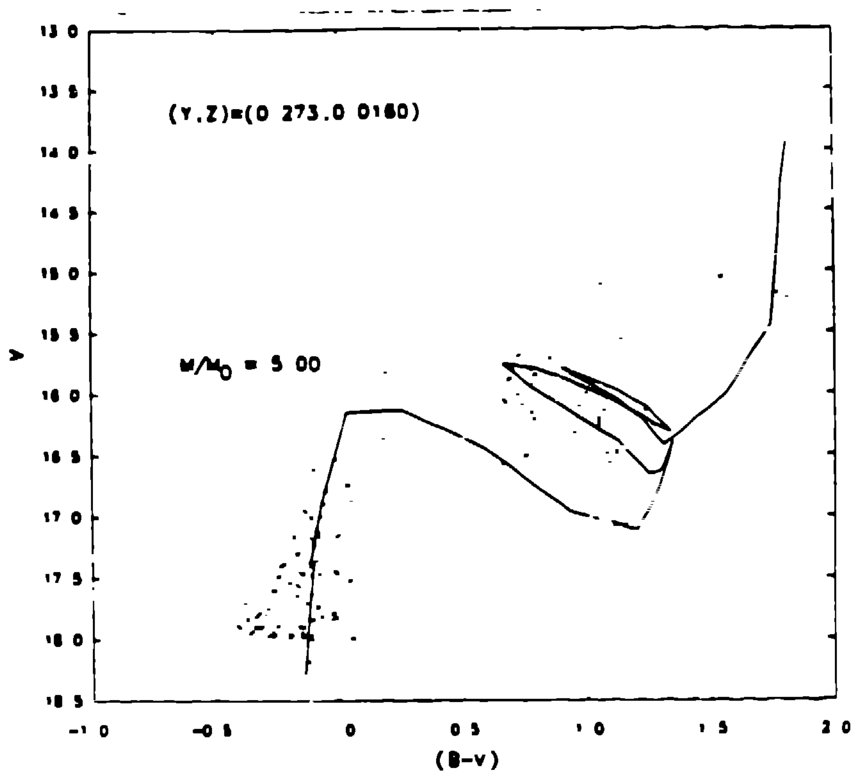


FIGURE 6. The observed color-magnitude diagram for NGC 1866 (dots) with the evolutionary track for a 5 $M_\odot$,(Y,Z) = (0.273, 0.015) model superimposed. The observed Cepheids are near the blue loop tip.

4 SUMMARY

In the review, the behavior of stellar evolution models relevant to Cepheid variables of Types I and II is discussed. Uncertainties in the theoretical models are examined, but are found not to be of such size as to seriously limit the usefulness of the theoretical predictions. Finally, observations and theoretical predictions are made for a number of cases to show areas of agreement and disagreement.

This work was performed under the auspices of the U.S. Department of Energy Contract # W-7405-ENG. 36.

Arp, H. (1967) Ap. J., 149, 91-106.
Becker, S. A. (1981a) Ap. J. Suppl. 45, 475-505.
Becker, S. A. (1981b) Ap. J., 248, 298-310.
Becker, S. A. & Cox, A. N. (1982) Ap. J., 260, 707-715.
Becker, S. A. & Iben, I. Jr. (1979) Ap. J., 232, 831-853.
Becker, S. A. , Iben, Icko Jr., & Tuggle, R. S. (1977) Ap. J., 218, 633-653.
Becker, S. A. & Mathews. S. J. (1983) Ap. J., 270, 155-168.
Bohm-Vitense (1984) In Observational Tests of the Stellar Evolution Theory, IAU Sym., 105, ed. A. Maeder, in press, Dordrecht: D. Reidel Publ. Co.
Brunish, W. M. & Truran, J. W. (1982a) Ap. J., 256, 247-258.
Brunish, W. M. & Truran, J. W. (1982b) Ap. J. Suppl., 49, 447-468.
Carson, T. R. & Stothers, R. (1976) Ap. J., 204, 461-471.
Cox, A. N. (1980) Ann. Rev. Ast. Ap., 18, 15-41.
Cox, A. N., & Tabor, J. E. (1976) Ap. J. Suppl., 31 27.-3?2.
de Loore, C. (1980) Spc. Sci Rev., 26, 113-155.
Despain, K. H. (1981) Ap. J. 251, 639-653.
Eggen. O. J. (1983) Aston. J., 88, 386-403.
Endal, A. S. & Sofia, S. (1976) Ap. J., 210, 184-19?
Endal, A. S. & Sofia, S. (1978) Ap. J., 220, 279-290.
Endal, A. S. & Sofia, S. (1979) Ap. J., 232, 531-540.
Evans, N. R. (1980) Bull. AAS., 12, 862.
Fernie, J. D. (1979) Ap. J., 231, 841-845.
Fernie, J. D. (1984) In Observational Tests of the Stellar Evoluti n Theory; IAU Sym 105, A. Maeder in press, Dordrecht: D. Reidel Publ. Co.
Forbes, J. E. (1968) Ap. J., 153, 495-510.
Fowler, W. A. (1984), private communication.
Fowler, W. A., Caughlan, G. R. & Zimmerman, B. A. (1967) Ann. Rev. Astr. Ap., 5, 525-570.
Fowler, W. A., Caughlan, G. R. & Zimmerman, B. A. (1975) Ann. Rev. Astr. Ap., 13, 69-111 (FCZII).
Fricke, K., Stobie, R. S., & Strittmatter, P. A. (1971) M.N.R.A.S., 154 ,23-46.
van Genderen, A. M. (1983) Astr. Ap., 124, 223-235.
Gingold, P. A. (1974) Ap. J., 193, 177-185.
Harris, M. J., Fowler, W. A., Caughlan, G. R., & Zimmerman, B. A. (1983) Ann. Rev. Astr Ap., 21, 165-176 (HFCZIII).
Henyey, L. G., Vardaja, M. S., & Bodenheimer, P. (1965) Ap. J., 142, 841-854.
Hirshfeld, A. W. (1980) Ap. J., 241, 111-124.
Hoppner, W., Kahler, H., Roth, M. L., & Weigert, A. (1978) Astr. Ap., 63, 391-399.
Huang, R. Q., & Weigert, A. (1983) Astr. Ap., 127, 309-312.
Iben, I. Jr. (1972) Ap. J., 178, 433-440.
Iben, I. Jr. (1974a) Ann. Rev. Ast. Ap., 12, 215-256.
Iben, I. Jr. (1974b) In Stellar Instability and Evolution, IAU Sym. 59, ed. P. Ledoux, A. Noels, & A. W. Rogers, pp. 3-34. Dordrecht: D. Reidel Publ. Co.

Iben, I. Jr. (1982) Ap. J., 260, 821-837.
Iben, I. Jr. & Renzini, A. (1983) Ann. Rev. Ast. Ap., 21, 271-342.
Iben, I. Jr., & Rood, R. J. (1970) Ap. J., 161, 587-617.
Iben, I. Jr. & Tuggle, R. S. (1975) Ap. J. 197, 39-54.
Johnson, H. R. & Whitaker, R. W. (1975) M.N.R.A.S., 173, 523-526.
Kettner, K. U. et. al. (1982) Z. Phys. A - Atoms and Nuclei, 308, 73-94.
Kippenhahn, R., Meyer-Hofmeister, E., & Weigert, A. (1970), Astr. Ap., 5, 155-161.
Kraft, R. P. (1970) In Spectroscopic Astrophysics, ed. G. H. Herbig, pp. 385, Berkeley: Univ. of California Press.
Lambert, D. L. (1984) In Observational Tests of the Stellar Evolution Theory, IAU Sym., 105, ed. A. Maeder, in press, Dordrecht: D. Reidel Publ. Co.
Lamers, H. J. G. L. M. (1981) Ap. J., 245, 593-608.
Lauterborn, D., Refsdal, S., & Weigert, A. (1971) Astr. Ap., 10, 97-117.
Lauterborn, D., & Siquig, R. A. (1974) Ap. J., 191, 589-592.
Luck, R., & Lambert, D. L. (1981) Ap. J., 245, 1018-1034.
Madore, B. T. (1977) M.N.R.A.S., 178, 505-511.
Maeder, A., & Mermilliod, J. C. (1981) Astr. Ap., 93, 136-149.
Magee, N. H., Merts, A. T., & Huebner, W. T. (1984) Ap. J., in press.
Matraka, B., Wassermann, C., & Weigert, A. (1982) Astr. Ap., 107, 283-291.
Mengel, J. G., Sweigart, A, V., Demarque, P., & Gross, P. S. (1979) Ap. J. Suppl., 40, 733-791.
Meyer-Hofmeister, E. (1972) Astr. Ap., 16, 282-285.
Parenago, P. P. (1956) Perem. Zvezdy; 11, 236.
Payne-Gaposchkin, C. (1974) Smithsonian Contr. Ap., 16, 1-32.
Reimers, D. 1975, Mem. Soc. Roy. Sci. Liese 6th Ser., 8, 369.
Renzini, A. (1977), In Advanced Stages of Stellar Evolution, ed. P. Boivier and A. Maeder, pp. 151-283, Sauverny: Geneva Observatory.
Robertson, J. W. (1972) Ap. J., 177, 473-488.
Robertson, J. W. & Faulkner, D. J. (1972) Ap. J., 171, 309-315.
Sandage, A. & Tammann (1969) Ap. J., 157, 683-708.
Schlesinger, B. M. (1977) Ap. J., 212, 507-512.
Schmidt, E. G. (1984) Ap. J., in press.
Schonberner, D. (1979) Astr. Ap., 79, 108-114.
Simon, N. R. (1982) Ap. J., 260, L87.-L90.
Slettebak, A. (1970) In Stellar Rotation, IAU Coll. 4, ed. A. Slettebak, pp. 3-8, Dordrecht: D. Reidel Publ. Co.
Sreenivasan, S.R. & Wilson, W.J.F. 1978, Ap. Space Sci., 53, 193-216.
Sweigart, A. V. & Gross, P. G. (1976) Ap. J. Suppl. 32, 367-398.
Szabados, L. (1983) Astr. Sp. Sci., 96, 185-194.
Szabados, L. (1984) In "Observational Tests of the Stellar Evolution Theory"; IAU Sym 105, ed. A.Maeder, in press, Dordrecht: D. Reidel Publ. Co.
Wallerstein, G. & Cox, A. N. (1984) Publ. Ast. Soc. Pac., in press.